Function Spaces, Analysis and Approximation
NU, Astana
Abstracts

Estimates of the best M-term approximations of periodic functions in the anisotropic Lorentz-Zygmund space

Gabdolla Akishev (Kazakhstan Branch, Lomonosov Moscow University)

Abstract. In the talk, we will consider the anisotropic Lorentz-Zygmund space of periodic functions of many variables and the Nikol'skii–Besov class in this space. The order-sharp estimates are established for the best M-term trigonometric approximations of functions from the Nikol'skii-Besov class in the norm of another Lorentz – Zygmund space.

Triebel-Lizorkin spaces with generalized smoothness and strong summability problem

Sergei Artamonov (National Research University Higher School of Economics, Friedrich-Schiller-Universität Jena)

Abstract. TBA

Multipliers for a pair of Morrey spaces

Evgenii Berezhnoi (Yaroslavl State University)

Abstract. Let an ideal space X on R^n, an ideal space l of two-sided sequences with the standard basis $\left\{e^i\right\}$, a collection $\left\{B\left(0,2^i\right)\right\}$ of bolls and a collection of disjoint annuli $\left\{D_i = B\left(0,2^i\right) \backslash B\left(0,2^{i-1}\right)\right\}$ be given. By the global Morrey space $M^\tau_{l,X}$ (the approximation global Morrey space $\overline{M^\tau_{l,X}}$), we mean the set of all functions $f \in L^{1,loc}\left(R^n\right)$, for each of which the following norm is finite:

$\left\| f \mid M^\tau_{l,X} \right\| = \sup_{t\in R^n} \left\|\sum_{i=-\infty}^{\infty} e^i \right\| f(t+.)\chi\left(B\left(0,2^i\right) |X\| \mid l\|$

$\left(\left\| f \left| \overline{M^\tau_{l,X}} \right\| = sup_{t\in R^n} \| \sum_{i=-\infty}^{\infty} e^i \, \| f(t+.)\chi\left(D_i\right) |X\| \mid l\| \right)\right.$.

Based on a new approach for a wide class of global Morrey spaces, we give an exact description of the multiplier space between two Morrey spaces from this class. It is shown that in this case the multiplier space for a couple of Morrey spaces is an approximation Morrey space structurally constructed from the original spaces.

References

1. E.I. Berezhnoi, A discrete varsion of local Morrey spaces. Izvestiya RAN: Ser. Mat. 81 (2017), no. 1, 3-31 (in Russian). English transl.: Izvestiya: Mathematics. 81 (2017), no. 1, 1-28.

2. E.I. Berezhnoi, Multipliers for local Morrey spaces. Positivity: 5(2022) DOI 10.1007/s11117-022-00951-9.

3. E.I. Berezhnoi, Multipliers for global Morrey spaces. Positivity: 3(2023), p. 1-18 DOI:10.1007/11117-02300994-6.

Integral representations of functions and their applications

Oleg Besov (Steklov Institute)

Abstract. Integral representations of differentiable functions of many real variables will be given and their applications to proving the continuity and compactness of embeddings of various spaces of differentiable functions and estimating the entropy numbers of embedding and differentiation operators will be described

On estimates of non-increasing rearrangement of generalized fractional maximal function

Nurzhan Bokayev, Amiran Gogatishvili, Azhar Abek (L.N. Gumilyov Eurasian National University, Astana, Kazakhstan, Country Institute of Mathematics of the Czech Academy of Sciences)

Abstract. Let $\Phi : R_+ \to R$. The generalized fractional-maximal function $M_\Phi f$ is defined for the function $f \in E(R^n) \cap L_1^{loc}(R^n)$ by $(M_\Phi f)(x) = \sup_{r>0} \Phi(r) \int_{B(x,r)} |f(y)|dy$, where $B(x,r)$ is a ball with the center at the point $x \in R^n$ and radius r. In the case $\Phi(r) = r^{\alpha-n}, \alpha \in (0;n)$

we obtain the classical fractional-maximal function $M_\alpha f$. We give a sharp pointwise estimates of the non-increasing rearrangement of the generalized fractional maximal function $(M_\Phi f)(x)$ [1]. It is shown that the obtained estimate is more sharp than the inequality which is followed from the estimate for the generalized Riesz potential [2]. 1. Bokayev N.A., Gogatishvili A., Abek A.N. On estimates of non-increasing rearrangement of generalized fractional maximal function.Eurasian Math. J., Vol. 14 (3023), Number 2, pp. 13-23.

2. Bokayev N.A., Goldman M.L., Karshygina G.Zh.Cones of functions with monotonocity conditions for generalized Bessel and Riesz potentials. Mathem. Notes. 104 (2018), no. $3., 356-373$.

Interpolation theory methods for nonlinear operators

Victor Burenkov (RUDN)

Abstract. Interpolation theorems of Marcinkiewicz, Calderon, and Stein-Weiss for a wide class of nonlinear operators will be presented. These theorems are applicable , in particular, to homogeneous operators, for $1 < p < \infty$ and to nonlinear Urysohn-type operators.

Universal sampling discretization of integral norms and sparse sampling recovery

Feng Dai (University of Alberta)

Abstract. In this talk, I will report some advancements in sampling discretization and recovery. My primary focus will be on my joint work with E. Kosov, A. Prymak, A. Shadrin, V. Temlyakov, S. Tikhonov in this area. The central topic of discussion will be the challenge of discretizing L_p norm in a high-dimensional space. The goal is to establish two-sided estimates of the L_p norm defined with respect to a general probability measure, using a finite sum of function values. The uniform discretization approach applies universally to all functions in the space, ensuring that the points are independent of any specific functions within the space. I will provide estimates for the required number of points based on the dimension of the space.

Multiple Fourier series with partial-monotone coefficients

Mikhail Dyachenko (Lomonosov Moscow State University)

Abstract. There are a lot of different definition of monotonicity of the coefficients of trigonometric series $\sum$_{$n \geq 1$} a$_n$$e$e^{$inx$} theorem holds for 2 m/(m + k) < p < ∞. We also discuss the possibility of strengthening this result.

Algebraic Properties of Function Spaces for Mikusinski's Operational Calculus

Arran Fernandez (Eastern Mediterranean University)

Abstract. The operational calculus of Mikusinski was originally constructed in the space of continuous functions on the closed half-line, a function space that forms a rng under convolution and whose properties are rather well-known. This space is sufficient for solving ordinary differential equations of integer order, but other spaces are needed for partial differential equations or fractional differential equations. We consider the different types of function spaces needed for Mikusinski's operational calculus in different settings, particularly Dimovski's spaces of continuous functions modified by power-function singularities. Their

algebraic properties are also relevant, and we prove various results about the ideal structure of these rngs.

Refinement of the mean angle in the Fejes Tóth problem

Dmitry Gorbachev (Independent)

Abstract. The Fejes Tóth problem about the maximum E of the mean value of the sum of angles between 3D lines with a common center is considered. L. Fejes Toth conjectured that E = pi/3 = 1.047... This conjecture has not yet been proven. Fejes Tóth also showed that E < 1.256. Recently F. Fodor, V. Vígh, and T. Zarnócz proved that E < 1.178. Finally, D. Bilyk and R.W. Matzke found that $E < 1.110$. We refine this estimate using an extremal problem of the Delsarte type. The results in further development of this problem are also presented.

Sharp bounds for distribution of martingale transform of bounded functions

Valerii Ivanov (Tula State University, Lomonosov Moscow State University)

Abstract. We propose a Sturm theory about zeros of discrete polynomials similar to the Sturm theory about zeros of polynomials in eigenfunctions of the classical Sturm-Liouville problem on an interval. We consider real functions defined at integer points of the interval [0, q] with zero boundary conditions at points -1 and q+1. A point on an integer interval is a zero of discrete function if its value at this point is zero or the product of the function values at this point and the previous point is negative. On an integer interval, a discrete Sturm–Liouville problem with zero boundary conditions is defined, and a system of orthogonal eigenfunctions is indicated for it. Sturm's theory for polynomials based on them consists of two statements. An eigenfunction with number n on the integer interval [0, q] has exactly n-1 zeros. A polynomial with eigenfunction numbers from m to n has at least m-1 and at most n-1 zeros. Two examples of the application of this theory. The set of eigenfunctions forms a Chebyshev system on an integer interval, for which Haar's theorem on uniqueness and Chebyshev's theorem on the criterion for a polynomial of the best uniform approximation on an integer interval are valid. If any number of consecutive decreasing zeros are removed from a polynomial of an orthonormal system of polynomials on the interval [-1,1], then the coefficients of the expansion of the resulting polynomial in the orthonormal system are positive and monotonic. The positivity of the expansion coefficients was proven by another method in 2007 by H. Cohn and A. Kumar.

On integral representation of the Green's function of the Dirichlet problem for the Laplace equation

Tynysbek Kalmenov (IMMM)

Abstract. In the work of Kalmenov T.Sh., Otelbaev M.O. it was established that the function $u(x)$ satisfies the boundary condition only and only if

$$\Delta_y \tilde{q}(\xi, y) = 0$$

Trace of the potential of a simple layer on $\partial\Omega$ given by the form

$$\left(D_S^{-1}\nu\right)(x) = \int_{\partial\Omega} \varepsilon(x,\xi)\nu(\xi)dS_\xi$$

it is a completely continuous self-adjoint operator in $L_2(\Omega)$ and its kernel $\varepsilon(x,\xi), x, \xi \in \partial\Omega$ is expanded into the series

$$\varepsilon(x,\xi) = \sum_{|m|=1}^{\infty} \frac{e_m(x)e_m(\xi)}{\lambda_m},$$

where $e_m(x)$ - is a complete orthonormal system of eigenfunctions of the operator D_S^{-1} corresponding to the eigenvalues $\frac{1}{\lambda_m}$. It's easy to check that

$$D_S^{-1}e_m(x) = \frac{e_m(x)}{\lambda_m}, D_S e_m(x) = \lambda_m e_m(x)$$

We call the function $G(x,y) = \varepsilon(x,y) - g(x,y)$ the Green's function of the Dirichlet problem if the function

$$u(x) = \int_\Omega G(x,y)f(y)dy$$

Sampling discretization problem for Orlicz-type norms

Egor Kosov (Centre de Recerca Matemàtica)

Abstract. Sampling discretization problem for integral L_p norms aims to find a good replacement of a given continuous probability measure on some compact set K with a discrete one in such a way, that L_p norms with respect to the initial measure and with respect to the new discrete one are comparable on a given finite dimensional subspace of the space of continuous functions on K. In the talk we will discuss a modification of this important problem when in place of usual L_p norms, Orlicz-type norms are considered. New discretization results for such type of norms will be provided and the applications of these results to the problem of sampling recovery will be shown.

Extrapolation and Interpolation in Weighted Grand Morrey Spaces

Alexander Meskhi (Kutaisi International University)

Abstract. Rubio de Francía's extrapolation theorems for weighted grand Morrey spaces $M_w^{p),\lambda,\theta}$ with Muckenhoupt A_p weights w are established. The same problems is studied for weights beyond the Muckenhoupt classes. These results, in particular, imply one-weight inequalities for operators of Harmonic Analysis in these spaces for appropriate weights.

Complex interpolation and duality problems for two-weighted grand Morrey spaces $M_{v,w}^{p),\lambda,\theta}$ are studied as well. The latter result is applied for one-weight boundedness problems for various operators of Harmonic Analysis in the aforementioned spaces. The talk is based on the papers [1], [2], [3].

References [1] E. Gordadze, A. Meskhi and M. A. Ragusa, On some extrapolation in generalized grand Morrey spaces and applications to PDEs, Electronic Research Archive, 2023 (accepted). [2] A.Meskhi, Extrapolation in new weighted grand Morrey spaces beyond the Muckenhoupt classes, Journal of Mathematical Analysis and Applications, 2023, https://doi.org/10.1016/j.jmaa.2023.127181. [3] A. Meskhi, H. Rafeiro and T. Tsanava, Duality and interpolation for weighted grand Morrey spaces, Trans. A. Razmadze Math. Inst. 177 (2023), no. 1, 149-155.

Tractable s-widths in weighted Wiener spaces

Moritz Moeller (TU Chemnitz)

Abstract. The best m-term approximation has been a rather theoretical subject of study in approximation theory since its inception by Stechkin in 1955. Recently however Jahn, T.Ullrich and Voigtlaender have found some practical application for it by using it in a new bound on the sampling numbers. One important class of spaces where this bound can give an improvement over existing ones are weighted Wiener spaces. Motivated by this a new bound for the best m-term approximation in these spaces will be shown in this talk as well as a sharp asymptotic bound on the Gelfand widths of these spaces will also be provided, which are a natural lower would on the sampling widths. A main focus of our research was in the development of a tractable result that still give good bounds in the preasymptotic setting.

Estimates for the norm of the Hardy-type operator in operator ideals

Mariya Nasyrova (Computing Center FEB RAS)

Abstract. One of the ways to construct an operator ideal is related to the s-numbers of operators: the (quasi)norm of an operator in an operator ideal is defined as the (quasi)norm of the corresponding sequence of its s-numbers in some sequence space [1,2].

Let the parameters $1 < \max(r,s) \leq q < \infty, 1 < p < \infty$ and $v, u,\ \omega$ be non-negative weight functions. We consider a generalized Hardy-type operator $Tf(x) = \int_0^x u(t)f(t)v(t)dt,\ x > 0$, acting from a weighted Lorentz space $L_v^{r,s}(R^+)$into another $L_\omega^{p,q}(R^+)$.

We introduce two auxiliary sequences, depending only on parameters and weights, which enable us to express the criteria of boundedness and compactness of a given operator. The conditions under which a compact Hardy-type operator, acting in Lorentz spaces, belongs to operator ideals generated by sequences of s-numbers are considered. Estimates for the norm of the Hardy-type operator in these ideals in terms of integral expressions depending on parameters and the weight functions of the operator are obtained.

References

1. Pietsch, A., Operator Ideals, Deutscher Verl d Wiss, 1978.

2. Carl, B., Stephani, I., Entropy, compactness and the approximation of operators, Cambridge: Cambridge Univ. Press, 1990.

3. Lomakina, E.N., Nasyrova M.G., Estimates for the norms of the Hardy operator in operator ideals, Siberian Mathematical Journal, 2024 (to appear).

Sharp bounds for distribution of martingale transform of bounded functions

Mikhail Novikov (Steklov Institute)

Abstract. The talk will be devoted to the following problem. Consider a martingale φ with the limit value φ_∞ satisfying the equality $|\varphi_\infty| = 1$ almost surely and its martingale transform ψ. The task is to describe the distribution function of ψ_∞. Namely, we want to characterise as precisely as possible the set of non-negative functions $f\colon R \to R$ for which the value $Ef(\psi_\infty)$ is bounded from above by an absolute constant. We will show how to reduce the problem to computing the minimal separately concave function $B\colon \{(x,y) \in R \to R\colon x-1 \leq y \leq x+1\}$ with fixed values on the boundary of the domain. As a result, using this object, we will find an exhaustive description of the integral properties of the random variable ψ_∞.

Anisotropic net spaces and interpolation properties of integral operators

Erlan Nursultanov (Lomonosov Moscow State University, Kazakhstan branch and Institute of Mathematics and Mathematical Modelling)

Abstract. The work defines general anisotropic net spaces. These spaces generalize anisotropic Lorentz spaces. A criterion for the quasi-weak boundedness of an integral operator in Lebesgue spaces is obtained. New sufficient conditions for the boundedness of the integral convolution operator in Lebesgue spaces are obtained. New inequalities of Hardy and Littlewood type in Lorentz spaces are obtained. Interpolation theorems for integral operators in anisotropic Lorentz spaces are proven

Weighted Spaces of Besov. Embedding and Interpolation Theorems

Ademi Ospanova, Leili Kussainova, Gulnar Murat (Eurasian National University)

Abstract. In this work, scales of weighted spaces $X_p^s(\Omega; \rho, v_s)$ for functions in n dimensional domains with normalization of the type $l_p(s \geq 0, 1 < p < \infty)$ are introduced. In particular, scales of Sobolev spaces $W_p^m(\Omega; \rho, v_m)$ and Besov spaces $B_p^s(\Omega; \rho, v_s)$.

Unlike Tribel spaces, for the weight functions $\rho(x)$ and $v_s(x)(x \in \Omega)$, no smoothness conditions and uniform growth or decrease near the boundary of the domain are imposed. The construction of these spaces allows obtaining descriptions of interpolation spaces and pointwise multipliers for corresponding pairs of the spaces. In this work, descriptions of Petre's interpolation spaces are obtained.

Fractional Orlicz-Sobolev spaces

Luboš Pick (Charles University)

Abstract. We will survey recent results obtained jointly with A. Alberico (Napoli), A. Cianchi (Firenze) and L. Slavíková (Praha) on sharp embeddings of Orlicz-Sobolev spaces of non-integer order into other type of spaces. The target spaces will include in particular rearrangement-invariant spaces, Morrey spaces, Campanato spaces, spaces of uniformly continuous functions enjoying a uniform modulus of continuity, and more. We focus on the optimality of function spaces appearing in the embeddings. We shall point out some interesting dissimilarities in comparison with the classical theory of embeddings of Sobolev spaces of integer order.

Besov-type function spaces on the hypercube based on the half-period cosine system

Martin Schäfer (TU Chemnitz)

Abstract. In the setting of periodic functions, which can be modelled as functions on the hypertorus $T^d \cong [0,1]^d$, the classical Fourier system is the system of choice for many applications. Turning to non-periodic functions on $[0,1]^d$, this system is not so well-suited any more as exemplified by the Gibbs phenomenon at the boundary. Hence, in the setting of non-periodic functions, other systems have been considered. One such system is the half-period cosine system, which occurs naturally as the eigenfunctions of the Laplace operator under homogeneous Neumann boundary conditions. In this talk, we introduce and analyze associated function spaces of Besov-type, which generalize earlier concepts in this direction.

Several remarks on the B-spline basis condition number

Alexei Shadrin (Cambridge University)

Abstract. Uniform boundedness of the B-spline basis condition number $\kappa_{k,p}$ (of order k, for the L_p-norm) is one of the key features in spline theory. It guarantees stability of numerical calculations with splines, provides a good local spline approximation, ensures existence of a bounded interpolating spline projector for any partition, shows how small the kth derivative of any interpolant could be, and many other things.

In this talk, we discuss three conjectures of de Boor regarding this number made in mid 70s and in the 90s.

1) The first one was that, for all k and p, this number grows as 2^k. This seems to be correct for the max-norm, but most likely we have a bit slower growth $k^{-1/2p}2^k$ for $p < \infty$. (The current upper bound is $k2^k$ for all p.)

2) The second one was that the extreme case occurs for the partition with no interior knots (the so-called Bernstein knots). This was shown to be wrong for the max-norm by de Boor himself, we show that it is not the case also for large $p < \infty$.

3) And the third conjecture by de Boor was that "the exact condition of the B-spline basis may be hard to determine". For this one, our correcting guess is that "the exact condition of the B-spline basis will never be determined".

Sharp bounds for distribution of martingale transform of bounded functions

Maria Skopina (Saint Petersburg State University)

Abstract. Wavelets on the sets of M-positive vectors in the Euclidean space are studied. These sets are multivariate analogs of the half-line in the Walsh analysis.Following the ideas of the Walsh analysis,the space of M-positive vectors is equipped with a coordinate-wise addition. Harmonic analysis on this space is also similar to the Walsh harmonic analysis, and the Fourier transform is such that there exists a class of so-called test functions (with a compact support of the function itself and of its Fourier transform). Dual and tight wavelet frames consisting of the test functions are studied.

Strong and weak associated reflexivity of certain function classes

Vladimir Stepanov (Computing Center FEB RAS)

Abstract. The report provides an overview of recent results on the problem of describing associated and doubly associated spaces to functional classes that include both ideal and non-ideal structures. The latter include first-order two-weight Sobolev spaces on the positive semiaxis. It is shown that, unlike the concept of duality, associativity can be "strong"and "weak". At the same time, the doubly associated spaces are divided into three more types. In this context, it is established that the space of Sobolev functions with a compact supports has weakly associated reflexivity, and strongly associated with a weakly associated space consists only of zero. Weighted spaces of Cesaro and Copson type

have similar properties, for which the problem has been fully studied and their connection with Sobolev spaces with power weights has been established. As an application, the problem of the boundedness of the Hilbert transformation from the Sobolev space to the Lebesgue space is considered.

Hardy Spaces of Fractional Order

Dmitriy Stolyarov (St. Petersburg State University)

Abstract. I will introduce new spaces of measures that form a scale connecting the space of measures of bounded variation with the real Hardy class H_1. The main two facts we wish to "interpolate" are a version of the Sobolev embedding (the Riesz potential acts "poorly" on the space of measures and maps H_1 to the best possible dilation invariant Besov space as well as some more sophisticated spaces) and dimensionality (we can say nothing about the singularity of an arbitrary measure whereas all "measures" in the Hardy class are absolutely continuous). This leads to (seemingly new) spaces H_1^β depending $\beta \in [0, d]$, here d is the dimension of the ambient space; we have that H_1^0 is the space of measures and H_1^d is the classical Hardy class. We describe the action of the Riesz potential on these spaces (in the spirit of trace inequalities of D. Adams) and also prove that any measure $\mu \in H_1^\beta$ has lower Hausdorff dimension at least β (the latter bound is sharp).

The motivation for these studies comes from the authors' earlier independent work on Bourgain—Brezis inequalities and dimension estimates for measures satisfying PDE or Fourier analytical constraints. In particular, we know that any coordinate of solenoidal charge lies in H_1^1 and conjecture that any coordinate of the gradient of a BV function lies in H_1^{d-1}.

Improved Hardy inequalities on homogeneous groups

Durvudkhan Suragan (Nazarbayev University)

Abstract. We establish a new improvement of the classical L^p-Hardy inequality on the multidimensional Euclidean space in the supercritical case. Recently, in the work of Frank, Laptev, and Weidl, there has been a new kind of development of the one-dimensional Hardy inequality. Using some radialisation techniques of functions and then exploiting symmetric decreasing rearrangement arguments on the real line, the new multidimensional version of the Hardy inequality is given. Motivations and some consequences are also discussed. Joint work with P. Roychowdhury and M. Ruzhansky.

New uncertainty principle relations for Fourier transforms

Sergey Tikhonov (Catalan Institution for Research and Advanced Studies)

Abstract. TBA

Tensor Decompositions in Mathematics and Applications

Eugene Tyrtyshnikov (INM RAS)

Abstract. Tensor decompositions become a very popular tool for modelling data in many application problems. We discuss some still open issues about the rank-bounded sets for the canonical polyadic decomposition and new developments of cross-approximation approach to optimization problems with the tensor train model.

References

1. Tyrtyshnikov E., Tensor decompositions and rank increment conjecture, Russian Journal of Numerical Analysis and Mathematical Modelling, 25 (4), 239–246 (2020).

2. Zheltkov D., Tyrtyshnikov E., Global optimization based on TT-decomposition, Russian Journal of Numerical Analysis and Mathematical Modelling, 25 (4), 247–261 (2020).

Sampling recovery, discretization and projections in the uniform norm

Mario Ullrich (JKU Linz)

Abstract. We present recent progress on optimal approximation in the uniform norm based on function values, and how this relates to discretization and the theorem of Kadets and Snobar on norms of projections. We also discuss some implications for approximation in other norms.

On Haar frames in Besov and Triebel-Lizorkin spaces

Tino Ullrich (TU Chemnitz)

Abstract. We survey on recent results by G. Garrigos, A. Seeger and T. Ullrich on the Haar wavelet system in Besov and Triebel-Lizorkin spaces. In the first part of the talk we give sharp conditions for the parameter range where the Haar system is an unconditional basis. This settles an open question by H. Triebel. We also consider the parameter range in which unconditionality does not hold. Surprisingly, in a range of parameters up to smoothness $s = 1$ the spaces $F^s_{p,q}$ and $B^s_{p,q}$ are characterized in terms of doubly oversampled Haar coefficients, the Haar frame coefficients. As a consequence, we obtain by bootstrapping that in case $1/p < s < 1$ the discrete Haar coefficient norm is equivalent to the standard $B^s_{p,q}$-norm. At the endpoint case $s = 1$ and $q = \infty$, we show that such an expression gives an equivalent norm for the Sobolev space W^1_p if $1 < p < \infty$, which is related to a classical result by Bockarev. Finally, in various endpoint cases we clarify the relation between dyadic and standard Besov and Triebel-Lizorkin spaces. This is joint work with G. Garrigos and A. Seeger.

On the approximation of vector-valued functions by samples

Andre Uschmajew (University of Augsburg)

Abstract. The approximation of a vector-valued function by a k-dimensional subspace plays an important role in dimension reduction techniques, such as reduced basis methods. For practical reasons, the linear subspace is often restricted to be spanned by samples of the function. By extending a well-known result on column subset selection for matrices, we show that for functions from L2 Lebesgue-Bochner spaces there always exist k sample points such that the resulting subspace approximation error is optimal up to a mild constant.

The study of Riemann-Liouville operators in weighted function spaces by splines

Elena. P. Ushakova (V.A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences)

Abstract. Estimates are obtained for the fulfillment of inequalities relating the norms of images and pre-images of RiemannLiouville operators I_α in weighted Besov spaces. The results are applied to the study of characteristic numbers of I_α and to the problem of boundedness of the Hilbert transform. Emphasis will be placed on spline wavelet systems, which were used as a key tool in solving the problems. The content of the talk is based on the publications below.

[1] Ushakova E. P., Ushakova K. E. Localisation property of Battle-Lemarié wavelets' sums // J. Math. Anal. Appl. 2018. - V. 461, No. 1. - P. 176-197.

[2] Ushakova E. P. Spline wavelet bases in function spaces with Muckenhoupt weights // Rev. Mat. Complut. - 2020. V. 33. - P. 125-160.

[3] Ushakova E. P. Spline Wavelet Decomposition in Weighted Function Spaces // Proc. Steklov Inst. Math. - 2021. V. 312. - P. 301-324.

[4] Ushakova E. P. The Images of Integration Operators in Weighted Function Spaces // Sib Math J. - 2022. - V. 63 . P. 1181-1207.

[5] Ushakova E. P. Ushakova K. E. Norm inequalities with fractional integrals // Algebra i Analiz. - 2023. - V. 35, No 3. - P. 185-219.

[6] Ushakova E. P. Boundedness of the Hilbert transform in Besov spaces // Analysis Math. - 2023. - V. 49. No 4. P. 1137-1174.

Global Cauchy problem for nonlinear evolution equations in super-critical spaces

Baoxiang Wang (Jimei University)

Abstract. We survey our recent works on the Cauchy problem for a class of nonlinear evolution equations including semi-linear heat, nolocal NLS, Navier-Stokes, NLKG equations in super-critical function spaces $E^{\sigma,s}$ for which their norms are defined by

$$\|f\|_{E^{\sigma,s}} = \left\| \langle\xi\rangle^\sigma 2^{s|\xi|} \widehat{f}(\xi) \right\|_{L^2}, s < 0, \sigma \in R.$$

Any Sobolev space H^κ can be embedded into $E^{\sigma,s}$, i.e., $H^\kappa \subset E^{\sigma,s}$ for any $\kappa, \sigma \in R$ and $s < 0$. We show the global existence and uniqueness of the solutions for those equations if the initial data belong to some $E^{\sigma,s}$ and their Fourier transforms are supported in the first octant, the smallness conditions on the initial data in $E^{\sigma,s}$ are not required for the global solutions.

Hardy–Littlewood-type theorems for multi-dimensional Fourier transforms

Ferenc Weisz (Eötvös Loránd University)

Abstract. We obtain Fourier inequalities in the weighted L_p spaces for any $1 < p < \infty$ involving the Hardy-Cesàro and Hardy-Bellman operators. We extend these results to product Hardy spaces for $p \leq 1$. Moreover, boundedness of the HardyCesàro and Hardy-Bellman operators in various spaces (Lebesgue, Hardy, BMO) is discussed.

Ball Banach Function Spaces Meet BBM, BVY & BSVY Formulae

Dachun Yang (Beijing Normal University)

Abstract. The concept of ball quasi-Banach function (BQBF) spaces was introduced in 2017 by Y. Sawano, K.-P. Ho, D. Yang and S. Yang. It is well known that some well-known function spaces, such as Morrey spaces, weighted Lebesgue spaces, mixed-norm Lebesgue spaces, and Orlicz-slice spaces, are ball quasi-Banach function spaces, but not quasi-Banach function spaces. In this talk, we will first recall the celebrated formulae of J. Bourgain, H. Brezis, and P. Mironescu, and the recent surprising formulae of H. Brezis, A. Seeger, J. Van Schaftingen, and P.-L. Yung. Then we will introduce some recent extensions of these formulae to Sobolev spaces associated with ball Banach function spaces. In particular, we will introduce some methods on how to overcome the difficulties caused by the deficiency of the translation invariance, the rotation invariance, and the explicit expression of the quasi-norm of BQBF spaces.

Hypoelliptic functional inequalities and ground states for higher order nonlinear Schrödinger type equations the approximation of vector-valued functions by samples

Nurgissa Yessirkegenov (SDU University)

Abstract. In this talk we will give a review of our recent research on hypoelliptic functional inequalities. As applications, we show the existence of ground state solutions to higher order nonlinear Schrödinger type equations. Moreover, we express the best constants in Sobolev and interpolation inequalities in the variational form as well as in terms of the ground state solutions. If time permits, we will discuss versions of these results in the settings of general (non-unimodular) Lie groups.

Some progress of pointwise multipliers on Besov spaces

Wen Yuan (Beijing Normal University)

Abstract. This talk is devoted to giving an introduction on the history and the progress of pointwise multipliers on Besov spaces. We first recall a series of known characterizations of the spaces of pointwise multipliers on Besov spaces. Then we present some of our recent works which describe pointwise multipliers on Besov spaces in some endpoint cases.

On fractional inequalities on metric measure spaces with polar decomposition

Gulnur Zaur (IMMM)

Abstract. In the presentation, we present the fractional Hardy inequality on polarisable metric measure spaces. The integral Hardy inequality for $1 < p \leq q < \infty$ is playing a key role in the proof. Moreover, we also show the fractional Hardy-Sobolev type inequality on metric measure spaces. Logarithmic Hardy-Sobolev and fractional Nash type inequalities on metric measure spaces are presented. In addition, we present applications on homogeneous groups and on the Heisenberg group.